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Teaching Statistics coherently within the NSW Mathematics syllabi for Years 7-12, and a proposal for doing it yet better

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Outline

As a statistician, I'm pleased to see in the NSW Board of Studies Mathematics syllabi much about the benefits that a knowledge of statistics can bring to the student. But I find no clear distinction maintained between statistical knowledge as a contribution to numeracy for everyone and statistical knowledge as a technical grounding for the few, namely, professionals working in quantitative fields. I believe that, when teaching any particular topic in statistics, the teacher should always be aware which of these benefits is his/her uppermost teaching goal, and should design his/her approach appropriately. To underline the distinctness of the goals is my first theme in this presentation:

1. What the study of Statistics can contribute to the education of school students.

There are, in the Mathematics syllabi, many theoretically interesting and practically useful themes in Statistics and its preliminaries – familiarity with data and with probability. But it seems to me that some of these themes overlap repetitiously across successive years, whilst important connections among others are not remarked on. Nor is there much to be read in the syllabi about linkage between statistical and mathematical topics, that might valuably be brought out in teaching. To say something constructive about these issues is my second matter:

2. How insightful teaching can make Statistics components coherent with each other and with 'Pure' Mathematics topics in the present Years 7-10 Mathematics syllabus and the proposed Years 11-12 Mathematics syllabi.

While the printed syllabi define a reasonable coverage of basic statistical ideas, consonant with the teaching hours allocated, students' ultimate advance in statistical ideas and methods over six years appears rather less extensive than their progress (on the calculus-based track) in algebra and calculus. For students who find themselves drawn, intellectually or affectively, to Statistics – and I suspect there will be quite a number – we should work towards offering something that can be more rewarding. So, lastly:

3. Why a Year 12 Statistics course (alternative to Mathematics Extension 1 or Extension 2), would be an excellent idea.

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1. What the study of Statistics can contribute to the education of school students.

I see two substantial benefits. Each is a worthwhile goal for classes to pursue.

(a) NUMERACY: a grasp of basic statistical ideas can help make *every* student more numerate.

By *numeracy* (sometimes called *quantitative literacy*) I mean a competence
(i) to understand, and argue back to, a *description* of the real world in terms of numbers, and
(ii) to draw accurate meaning from, and criticise, an *argument* about the real world expressed in numbers.

Descriptions and arguments relating to real world contexts commonly involve uncertainty. If the uncertainty is acknowledged, it generally means we are dealing with *statistical* descriptions and arguments. We are assailed by them from all sides nowadays – from the framing of serious public policy debates (e.g. on housing affordability or the evidence for climate change) to the subtle persuasions of product marketers. In our relatively innumerate society, people too often accept uncritically statistics presented by those with vested interests. Advancing statistical literacy in our society is a pressing need. It deserves our committed efforts.

The topics of data and data analysis, present in the Years 7-10 Mathematics syllabus, are an appropriate initial setting for developing statistical literacy. Indeed, identifying misrepresentations of data is an explicit focus of the syllabus. I believe that teaching, during these four years, that concentrates on enhancing numeracy and is rich in examples, can prove pivotal for increasing both student appreciation of and student interest in the practical benefits that accrue. This will be so all the more if collegial participation makes it a reality in class work across the curriculum.

(b) OPTIMAL DECISION-MAKING UNDER UNCERTAINTY: knowing something in technical detail about how methods of statistical inference serve as practical guides for making optimal decisions in the face of uncertainty (so characteristic of the real-world) gives mathematically-inclined students a grounding that can be valuable to them in subsequent study of any statistics-using field – and, as we know, such fields are legion! It is, I believe, a goal best pursued through Years 11-12.

This is not a novel insight: it is already implemented in senior-year Mathematics syllabi both interstate and overseas. Yet it's a proposition only now being recognised locally. Happily, an introduction to the inferential topic of confidence intervals is proposed in the draft Mathematics Advanced syllabus.

The above two goals for statistics education are distinct, and students will be best served by teaching that is appropriate to its goal.

But some attributes of good teaching (and of a good syllabus) are equally important for either goal. I want to highlight in this regard the attribute of intellectual *coherence*. To see just what I mean by 'coherence' I'll refer you to my online paper, Sowey (1995), which explores salient factors that make for long-term retention of student learning. I trust it is self-evident that lack of coherence, whether in the syllabus or in classroom presentation, will be an obstacle to good learning. Whether the 'data' and 'probability' topics in the Year 7-10 and the Year 11-12 syllabi represent a coherent sequence through the successive years, and whether these two topics have coherent links with other mathematical topics, year by year, is a matter that deserves some closer examination.

While I'm speaking of 'coherence', I should also mention 'perspective'. A perspective view of any subject matter is one from a vantage point above the fine detail of that subject matter. It's from a perspective view that the coherence of any subject matter, and of its teaching, can best be appreciated. (This, too, is discussed in my 1995 paper, just mentioned). Jointly, coherence and perspective reveal the *structure* of the subject matter and of its teaching. And perceiving structure is, I hope you will agree, an utterly indispensable preliminary to sound comprehension and retention of learning.

Let me come to my second matter:

2. How insightful teaching can make Statistics components coherent with each other and with 'Pure' Mathematics topics in the current Years 7-10 Mathematics syllabus and the proposed Years 11-12 Mathematics syllabi.

Through Years 7-10, the Mathematics syllabus is strong on 'data' and on 'probability', the joint foundations on which the principles of statistical inference rest. But in this syllabus these two topics remain essentially disjoint from one other, and indeed this separation continues, in the proposed Mathematics General 1 and 2 and Mathematics Advanced syllabi, through Years 11-12. It's true that the *word* 'statistics' appears here and there under the topic 'data' or 'data analysis', but there appears to be no prompting in the syllabus documents to urge that, in teaching 'data' and 'probability', these topics be explicitly linked, say in Year 10 – at the very least to provide a perspective over their eventual integration in the study of statistical inference (whether that will be at school or at tertiary level). How much more meaningful it would be for a student in Year 12 Mathematics Advanced if he/she had been aware for some time that the theory of confidence intervals is a 'natural' outgrowth of the wish *to go beyond* just the information in a given set of data, and to infer something about the population from which the given data are a sample – *and was actually expecting to learn how this may be achieved*. Of course, insightful teachers may already be providing such perspectives, but I can comment only in relation to the evidence I have, namely what I see in the printed syllabi.

To give an idea of the kind of integrating perspective I have in mind, I'm attaching to this handout a diagrammatic overview of the syllabus that I issued to my students early in the first year statistics course Quantitative Methods B at UNSW. I returned to this in class at several stages during the semester. There is rather more technical detail in this diagram than is relevant at high school level, but I trust it will make my proposal more concrete.

In tracing through the links in such a perspective, there is opportunity to bring out the fact that statistical inferences are inductive inferences, and this, in turn, permits a discussion of inductive logic. Has such a discussion any point at high school? I would say yes, indeed, though it is probably best left to Year 12. My encouragement to teachers to offer a logic perspective prior to teaching the 'confidence intervals' topic in the proposed Mathematics Advanced course stems from my view that it is very valuable for students to understand the relationship between mathematical inference and statistical inference – apparently alike yet essentially distinct, coherent yet detached. The fundamental distinction lies, as you will know, in the difference between deductive logic and inductive logic.

It's a pity that no time has been allocated in the proposed new syllabus for even a skeletal comparative account of these systems of logic. It seems, in fact, that the *meaning* of 'deductive logic' is not a syllabus item in *any* of the Mathematics syllabi. Unless teachers are proactive on this account, students may go right through their school mathematics studies without learning either the name or the strengths and limitations of the logic that underpins most of their subject matter. Lest you think it too demanding to compare and contrast systems of logic at high school level, I would like to draw your attention to some of the material in the Queensland Year 11-12 course 'Philosophy and Reason' (Queensland Studies Authority (2004)) – which is bracketed with that State's Years 11-12 Mathematics courses – and to the logic perspective described in my paper Sowey (2005), which was designed for introductory statistics at university, but is readily adapted for senior high school.

And now my third matter:

3. Why a Year 12 Statistics course (alternative to Mathematics Extension 1 or Extension 2), would be an excellent idea.

Pages 38-41 of the *Draft Writing Brief for Mathematics Stage 6 Calculus-based Courses* (see www.boardofstudies.nsw.edu.au/syllabus_hsc/pdf_doc/maths-st6-calculus-dwb.pdf) sets out in some detail the way in which the content of the proposed Mathematics Advanced course and its two Extension courses has been arrived at. In order not to put what is educationally outstanding about the current calculus-based courses too much at risk, all change is incremental. The Board of Studies document explains: "[a]dditions to the content have been accommodated by removing content that does not build on Stage 5 syllabus outcomes, or that is of a level of abstraction unlikely to result in the development of conceptual understanding." In this way, an opening has been created for a topic in statistical inference, namely, confidence intervals – allocated 12 hours (that is, 10%) of the teaching time assigned to Mathematics Advanced. The document gives the justification for this innovation: "Statistical techniques allow professionals in almost all fields to analyse, interpret and make predictions in relation to the large volumes of data. ... Statistical literacy... has become increasingly important in a wide range of professions. An introduction to aspects of 'non-deterministic' thinking will assist students' transition to undergraduate courses requiring Statistics and/or Data Analysis." Perhaps one shouldn't cavil at what is certainly a welcome step forward, but, from a statistician's point of view, the new statistical material is a less-than-substantial advance, in light of the proffered justification.

I've touched on this concern in discussing my first and second matters, above. Essentially, it is that the data, probability and statistical topics, as they appear in the various Mathematics syllabi for Years 7-11, do not appear to me to be focused for eventual integration in the Year 12 Mathematics Advanced course. To put it another way, the building blocks have been delivered but the architect's plans are not to hand. This, then, lays a heavy responsibility on the teacher to devise the appropriate structure, for (as I've already remarked) structure is indispensable to the student for sound comprehension and retention of learning.

To compound this challenge to the teacher, I should add that, to a practising statistician, there is good reason to teach Statistics (and statistical inference, in particular) apart from Mathematics. Let me say something about why many practising statisticians view Statistics as not a part of Mathematics.

Granted, Mathematics and Statistics have aspects in common. At a basic practical level, they both involve numerical evaluations of formulae. Further, a part of each discipline is devoted to theorem proving in the abstract, and a part is devoted to modelling the real world and making practical inferences on the basis of these models. In both disciplines the theorems (of 'pure mathematics' and of 'mathematical statistics') are certain (given the truth of the axioms that underpin them) because the logic used in establishing them is deductive logic.

Where they differ is in their models of the real world. Mathematical models are *deterministic*, that is, either they refer to situations of certainty, or otherwise they disregard the elements of randomness. Statistical models, by contrast, are *stochastic*, that is, they relate characteristically to probabilistic situations and explicitly take into account the elements of randomness.

Since uncertainty is intrinsic to and pervasive in real world affairs, deterministic models are likely to serve less well than stochastic models for making inferences about the real world from a given set of data. Specifically, that is because, for deterministic models describing chance situations, we have no way of reliably quantifying the probability that mathematical inferences we make from them are accurate.

The same is not the case for almost every kind of statistical inference from stochastic models: we *do* have a way of reliably assessing the probability that our inferences from such models are accurate. We have, moreover, a set of rules for optimising this probability. This set of rules is the system of *inductive logic*. And the system of inductive logic is none other than the logic of Statistics. Thus, Statistics is the discipline that is directly concerned with producing optimal inductive inferences.

I can summarise like this. The intellectual value of Mathematics is the certainty of its deductive inferences in deterministic contexts. However, its inferences are unassessable for accuracy in stochastic contexts. By contrast, the intellectual value of Statistics is the assessability for accuracy of its inductive inferences in stochastic contexts. But it has nothing unique to contribute in deterministic contexts. (I hope this isn't too esoteric for non-statistician readers.)

So when practising statisticians view Statistics as not a part of Mathematics, they are highlighting the unique value of its inductive inferences from quantitative data in contexts of uncertainty. This is something which Mathematics cannot offer.

Let me assure you that, behind this view on the part of many statisticians, I detect no ill-will whatever towards Mathematics, mathematicians, or mathematics teachers!

What this analysis leads to is, firstly, a point I have made already – but it will do no harm to repeat it. It's the high desirability that students of Mathematics Advanced become aware *during their Year 12 studies* of the logical contrast between Mathematics and Statistics.

And secondly, the desirability of offering Statistics as a course of its own at Year 12 level.

I would, therefore, recommend that the Statistical Society of Australia, with (I hope) the support of the Mathematical Association of NSW, approach the NSW Board of Studies with a proposal to introduce a 60 hour Year 12 course in Applied Statistics, as an alternative to Mathematics Extension 1 (or Extension 2, for students who wish to take Extension 1). If there is active support for this initiative in the coming year, then I believe that the new course could realistically be introduced in 2012.

I acknowledge the view expressed by the MANSW (in its written submission to the Mathematics Stage 6 Review, see www.mansw.nsw.edu.au/members/stage-6-review/MANSW-Submission.pdf) that many schools may not have the teachers or the resources available to implement an additional Year 12 Mathematics unit. Should this inhibit the initiative, or should schools be prepared eventually to select among Year 12 Mathematics options in the same way as they currently choose which foreign languages they will offer?

I also acknowledge the caution by the Board of Studies (in its Broad Directions for the Mathematics Stage 6 Review, see www.boardofstudies.nsw.edu.au/manuals/maths_ s6_rev_ dev_2006.html) that, if there is to be further Statistics content in Stage 6, it is essential to address "implications in relation to the extent of relevant teacher expertise [and] professional development...". Clearly, a need for professional development of teachers is to be expected. This, too, should, I believe, be seen by the Statistical Society of Australia as an opportunity for constructive contribution of its members' expertise.

I have titled the Statistics course I am proposing *Applied Statistics* to signal that I do not envisage it as a course in elementary *mathematical* statistics, that is, its principal raison d'être is not to display the mathematical infrastructure of statistical methods. Its rigour will come not from study of proofs of theorems but, rather, from the coherence of its subject matter and its call on disciplined thinking to keep constantly in view and in balance a myriad practical issues. Neither do I see Applied Statistics as composed of a catalogue of statistical *methods* in search of employment. In Applied Statistics, statistical methods will be shown as developing from the kinds of data that statisticians encounter and from the kinds of questions that statisticians (and others!) ask of their data. I would expect there to be as much exposition of the qualitative aspects of statistical practice and of the meaning of statistical conclusions as there is of analytical technique. And, naturally, discussion of the nature of deductive logic and inductive logic would not be passed over!

Which specific topics should be included, and how should they be arranged? That is for consideration on another occasion.

There are many precedents to guide us. By this I mean that there are today many school systems in which a Statistics subject at the level that I am envisaging (or substantial components of such a subject) are already successfully in operation. To name but two, I mention the International Baccalaureate (IB) final year Mathematics courses and the US Advanced Placement (AP) course in Statistics.

Syllabi for IB senior Mathematics courses on the IB website (www.ibo.org) are accessible by password, but that access seems to be restricted to schools that offer the IB program. However, on the open web there is an IB statistics syllabus outline at 'standard level' here: www.isparis.edu/documents/curriculum/IBdiploma/mathematicssl.pdf [See Topic 6], and an outline at 'higher level' here: www.cis.edu.hk/sec/math/ib/IBH.htm

The AP course syllabus is on the web at www.collegeboard.com/student/testing/ap/ sub_ stats.html?stats

References

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